



Impact of acoustic impedance and flow resistance on the power output capacity of the regenerators in travelling-wave thermoacoustic engines

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ABSTRACT

This paper considers the role of acoustic impedance, flow resistance, configuration and geometrical dimensions of regenerators on the power produced in travelling-wave thermoacoustic engines. The effects are modelled assuming a pure travelling-wave and ideal gas, which allows defining a pair of dimensionless factors based on the “net” acoustic power production. Based on the analysis provided, the acoustic power flow in the regenerators is investigated numerically. It is shown that impedance essentially reflects the proportion between the acoustic power produced from heat energy through the thermoacoustic processes and the acoustic power dissipated by viscous and thermal-relaxation effects in the regenerators. Viscous resistance of the regenerator mainly determines the magnitude of the volumetric velocity and then affects the magnitude of acoustic impedance. High impedance and high volumetric velocity are both required in the regenerators for high power engines. The results also show that the optimum transverse dimension of the gas passage exists, but depends on the local acoustic impedance. In principle, it is possible to obtain an optimum combination between these two parameters.

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1. Introduction

In broad terms, thermoacoustic effect refers to the energy transfer between a compressible fluid and a solid material in the presence of an acoustic wave. There are typically two practical implementations of the thermoacoustic effect: imposing an acoustic wave in the compressible fluid along a solid material leads to hydrodynamic heat pumping effects (due to the compression and displacement of the fluid elements), which in turn result in generation of a temperature gradient within the solid material in the direction of the sound wave propagation. Conversely, imposing an appreciable temperature gradient within the solid material may result in spontaneous generation of an acoustic wave along the direction of temperature gradient in its vicinity. These two interactions form the basis for engineering thermoacoustic coolers (or heat pumps) and thermoacoustic engines, respectively.

The thermoacoustic effect was first qualitatively explained by Rayleigh, who formulated the following criterion [1]: “If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged”. However, it was not until 1960s when a much more accurate quantitative theory was developed by Rott [2,3], who derived the wave equation and energy equation for a single frequency sound propagating along a temperature gradient in a channel.

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There are two basic designs of thermoacoustic engines implemented so far: standing-wave and travelling-wave. In the standing-wave engines [4–6], a thermoacoustic core (a thermoacoustic “stack” sandwiched between the hot and cold heat exchangers) is placed in a half-wavelength or quarter-wavelength standing-wave resonator. Imperfect heat transfer between the gas and the solid surface is used to introduce a significant time delay between the gas motion and the gas thermal expansion (or contraction) to meet Rayleigh’s criterion. By analogy with RLC electrical oscillators that dissipate much electrical power to sustain their oscillations, the standing-wave engines are fundamentally inefficient due to the imperfect heat transfer. The highest efficiency, defined as the ratio of acoustic power running out from the ambient heat exchanger over the heat input into the hot heat exchanger, reported in the open literature is only 18% [6]. Limited by the intrinsically irreversible thermodynamic cycle, the thermal efficiencies of standing-wave engines are thought to have an upper bound of about 20% [7].

From the point of view of thermodynamics, to improve the thermal efficiency of standing-wave engines, one has to improve the heat transfer between the gas and the solid surface. However, this contradicts the basic design feature of the standing-wave engines: that is their reliance on the imperfect heat transfer to achieve the required time delay to sustain the oscillation. Ceperley [8,9] was first to realize that when a travelling sound wave propagates through the regenerator (where the thermal contact between the gas and solid material is excellent) from the cold to the hot end, the heat transfer interaction between the gas and the solid

material undergoes a Stirling-like thermodynamic cycle: the gas experiences thermal expansion during the displacement toward the higher temperature and thermal contraction during the displacement toward the lower temperature (see Fig. 1a). In this way, the correct time phasing is obtained to meet the Rayleigh's

criterion to produce power. Strictly speaking, however, the thermoacoustic core (a regenerator sandwiched by the hot and cold heat exchangers) just works as a power amplifier. To complete this Stirling-like thermodynamic process, acoustic power has to be fed to the ambient end of the regenerator with a near travelling-wave

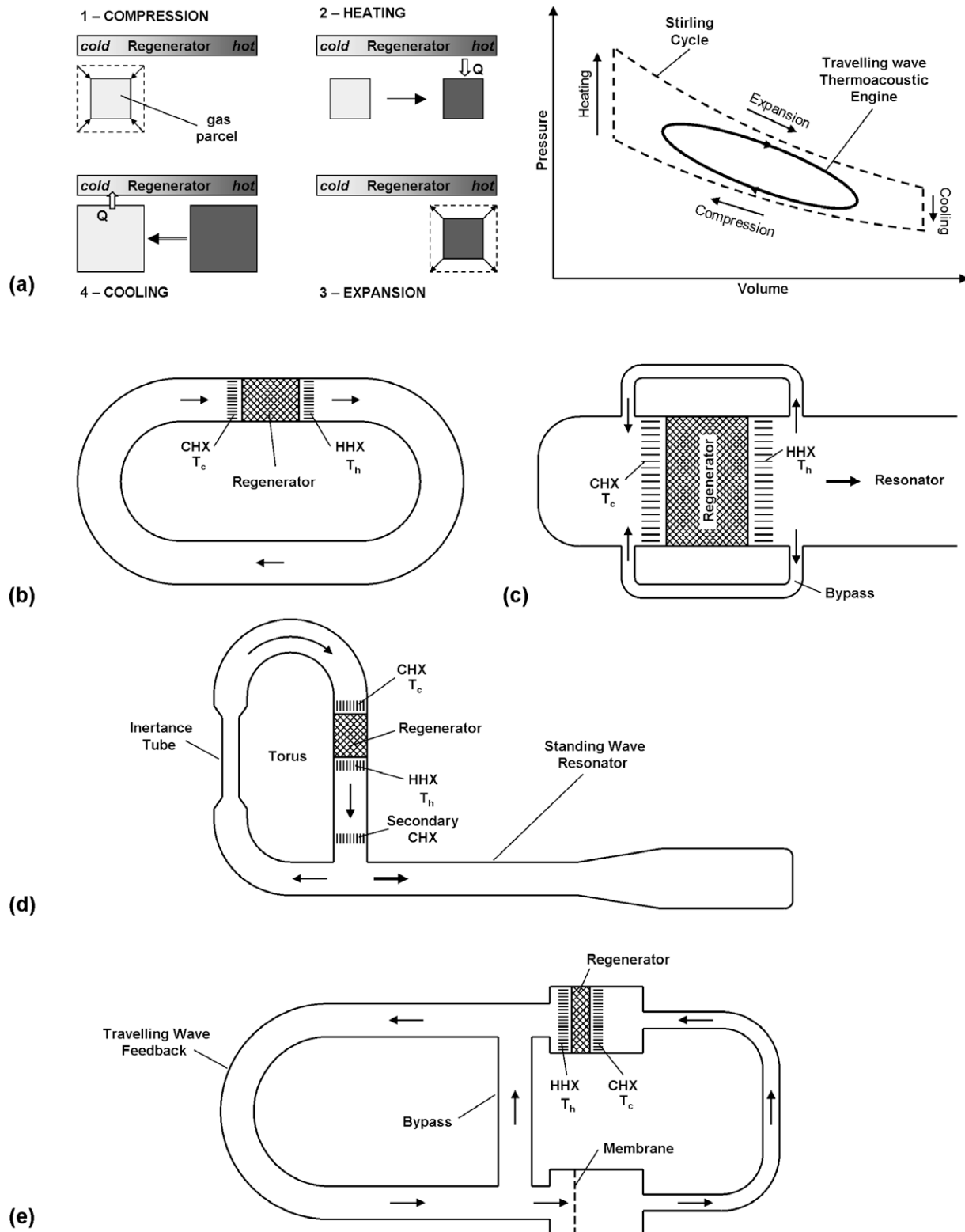


Fig. 1. Schematic diagrams of different types of travelling-wave thermoacoustic engines. The arrows show the directions of the acoustic power flow; (a) schematic of the thermodynamic cycle that a “gas parcel” experiences in the regenerator; (b) one-wavelength looped tube travelling-wave thermoacoustic engine; (c) bypass-type travelling-wave thermoacoustic engines; (d) torus-type travelling-wave thermoacoustic engine; (e) hybrid geometry with a travelling feedback.

phasing. Therefore, either an external acoustic source, or a feedback mechanism should be introduced. For example, Sugita et al. [10] used an electro-dynamic acoustic source to test the travelling-wave power amplifier for cryo-cooler applications. In Gardner and Swift's [11] cascade thermoacoustic engine, one stage of a standing-wave thermoacoustic core is used to produce acoustic power, which is fed into the adjacent travelling-wave stages for power amplification. Various acoustic feedback mechanisms were used in travelling-wave engine designs by Yazaki et al. [12], de Blok [13], and Backhaus and Swift [7,14].

Yazaki et al. [12] were probably the first to demonstrate a practical travelling-wave thermoacoustic engine, which is schematically shown as Fig. 1b. Here, the thermoacoustic core was placed in a one-wavelength "looped tube", which transports part of the acoustic power (out from the hot end of the thermoacoustic core) back to the ambient end. However, this engine proved very low efficiency, as later realized due to the very low acoustic impedance of the working gas in the regenerator, which caused large viscous losses resulting from high acoustic velocities in the regenerator and the resonator [12]. At almost the same time, two other different travelling-wave engines were proposed independently: de Blok [13] demonstrated a travelling-wave engine with a bypass feedback mechanism, which is schematically shown as Fig. 1c. Backhaus and Swift [7,14] proposed a different type, in which thermoacoustic core is placed within a torus with a length much shorter than the acoustic wavelength at the operating frequency. A long standing-wave resonator is connected to this torus just after the secondary ambient heat exchanger. It is schematically shown as Fig. 1d. The thermal efficiency of the latter (defined as the acoustic power delivered to the long resonator over the heat input) is so far the highest reported: 30%, equivalent to 41% of the theoretical Carnot efficiency.

Essentially, the torus (without the long resonator) of Backhaus and Swift's travelling-wave engine is an acoustic analogue of the free-piston Stirling engine. The main idea contained in their innovation is to mimic the free-piston Stirling engine using a compact acoustic network (i.e. the torus): using acoustic inertance, compliance and resistance. Its high efficiency was achieved by several new means: firstly, they used an "inert" feedback (inertance tube) to bring the timing phase close to zero in the regenerator. By analogy with RLC electrical oscillators, this mechanism added a positive feedback to the thermoacoustic power amplifier, thus, the self-oscillation can be sustained. Secondly, the compact network obtained a high acoustic impedance in the regenerator to suppress the dissipation: namely 15–30 times of $\rho_m a/A$, where ρ_m is the mean density of the gas, a is sound speed and A is the cross-sectional area. Thirdly, the thermal buffer tube (TBT) and the "jet pump" were utilized to suppress the Rayleigh streaming and Gedeon streaming which caused the parasitical heat losses. Finally, a long quarter-wavelength branch resonator was introduced to the torus, which provides the acoustic resonance to let the torus work at a relatively low frequency (compared with one-wavelength mode of the torus). As a result, the lengths of the parts in the torus are much smaller than the wavelength, which reduced the acoustic dissipation in the torus.

However, it should be noted that, the role of this long standing-wave resonator is as harmful as it is helpful. In essence, it is a Helmholtz acoustic resonator, its large surface dissipates much of the acoustic power delivered from the torus. If this dissipation is counted, the achievable thermal efficiency (useful acoustic power over the heat input) will be less than 0.30. In essence, this standing-wave resonator has two functions at the same time: a resonator to achieve the low frequency resonance for the torus and a parasitic (harmful) acoustic load. Theoretically, one should attempt to design a resonator which can sustain the resonance but with low harmful dissipation. Several

researchers tried different shapes and sizes of resonators to achieve this goal [15–17]. It was reported that the resonators of alternative form can improve the engine's performance in terms of the pressure amplitude. However, no improved thermal efficiency has been reported so far.

An alternative to using acoustic resonators is to introduce mechanical resonators into the thermoacoustic engine design. Sugita et al. [10] presented such an approach which used a pair of solid displacers as the resonator instead of a long resonator tube to realize both the standing- and travelling-wave thermoacoustic pressure generators. A self-sustained acoustic oscillation was obtained in their engine. Here, the displacers are functionally similar to those in free-piston Stirling engines. However, this solution is not practical due to the high cost of low-dissipation mass-spring oscillators (displacers mounted on flexure bearing), and due to adding moving parts, which were to be avoided in thermoacoustic engines in the first place.

An alternative strategy to reducing the harmful parasitic dissipation is to design a mechanism which can provide the resonance, but at the same time directly extract the acoustic power. Backhaus et al. [18] demonstrated such a travelling-wave thermoacoustic electric generator. Instead of the long resonator, they connected a pair of moving coil linear alternators to the torus. The mechanical resonance of the linear alternators induced and sustained an acoustic resonance in the torus, while the alternators utilized the acoustic power directly to produce electricity. After a careful optimisation, the engine efficiency (acoustic power to the alternator pistons over the heat input) of 24% and thermal-to-electric conversion efficiency (electricity power to heat input) of 18% were obtained.

The most recent development on the travelling-wave engines is the work by de Blok [19]. He pointed out that in the torus-type travelling-wave engine (see Fig. 1d), the combination of the high regenerator impedance and the large acoustic loss in the standing-wave resonator make the onset temperature difference very high. To reduce the onset temperature difference and apply this technology to utilize solar and waste heat, he proposed hybrid configuration with a travelling-wave feedback waveguide, which is schematically shown as Fig. 1e.

In fact, this new configuration can be looked upon as an improved variation of Yazaki's looped engine: (i) the phase shifters (inertance tube and compliance) are introduced to tailor the acoustic phasing to a near travelling-wave phasing at the cold end of the regenerator; (ii) a bypass is added to shunt part of the volumetric velocity, that is to turn it away from the engine core branch. In this way, a travelling-wave looped resonator (consisting of the feedback wave guide and the bypass tube) is achieved. In essence, the acoustic wave keeps running in this looped resonator with a near travelling-wave phase almost everywhere. The engine core branch extracts part of acoustic power from this looped resonator, and then tailors the acoustic phase to a near travelling-wave phasing and feeds it into cold end of the regenerator. After the amplification in the regenerator, the amplified acoustic power is injected back to the looped resonator. Because of the near travelling-wave phase, the acoustic loss in the looped resonator is quite low. Furthermore, the regenerator was deliberately designed with relatively low impedance and a large cross-sectional area. Both of these measures reduced the dissipation in the regenerator. As a result, the onset temperature difference was reduced significantly. The average onset temperature difference was reduced to 65 K, which is the lowest reported so far in the open literature.

The other advantage of de Blok's engine is that the "soft spot" (near travelling-wave phasing region) for the regenerator is relatively long, thus, multiple (i.e. multistage) thermoacoustic cores can be utilized in one engine. This would be very helpful for

increasing the power density when using low-temperature heat sources. However, although the relatively low regenerator impedance helps to obtain the low onset temperature difference, it is necessary to consider the regenerator impedance when applying this type of engine in high pressure amplitude situations when a high power density is required. On the other hand, the very short regenerator is a particular design for utilizing the low temperature difference. If one wants to apply this design to high temperature heat source to get high power density, the heat loss from the hot heat exchangers to the ambient heat exchangers (especially in multistage design) will be considerable. Unfortunately, so far, there is insufficient experimental data to prove that this new type of engine has a better efficiency than the torus type with a standing-wave resonator.

In parallel to the work on devising an appropriate acoustic network (the resonator designs discussed above), there have also been a number of studies focused on the optimisation of the regenerators for travelling-wave engines. Backhaus and Swift [20] fabricated and tested a parallel-plate regenerator, which has the dimensions similar to the tested stainless stacked-screen regenerator mentioned above [14]. They tested this parallel-plate regenerator using the same engine in which the stacked-screen regenerator had been tested. It has been reported that, using the parallel-plate regenerator and under similar test conditions, the engine delivered about 1750 W of acoustic power to its resonator [20], which was almost double the power delivered in the stacked-screen regenerator configuration [14]. This showed that the parallel-plate regenerator was clearly superior to the stacked screen one. The difference in regenerator performance was attributed to a nonlinear resistance in the stacked-screen regenerator at high pressure amplitude, however, no further detailed analysis of this difference was presented.

Yu et al. [21] carried out experimental investigations of the optimum transverse dimensions of the channel of the stacked-screen regenerator in a small-scale travelling-wave engine. The experimental results revealed that the ratio of δ_k/r_h plays an important role in controlling the onset of the engine. Here the thermal penetration depth $\delta_k = \sqrt{2k/\omega\rho_m c_p}$ (k is the thermal conductivity, c_p is the isobaric heat capacity per unit mass, ρ_m is the mean gas density and ω is the angular frequency). Using the lowest onset temperature difference as an indicator of regenerator performance, it has been shown experimentally that an optimum δ_k/r_h was between 4 and 7, when the hydraulic radius of the regenerator and the mean pressure vary in a large range (the investigations having been conducted for both helium and nitrogen). Furthermore, the experimental optimum δ_k/r_h increases slightly as hydraulic radius decreases. In fact, decreasing the hydraulic radius increases the viscous resistance in the regenerator. However, the impact of these parameters of the regenerator on the optimum transverse dimension was not investigated systematically.

Some similar work on optimising regenerators has also been carried out for coolers. The non-dimensional parameter $StPr^{2/3}/f_F$ was proposed to evaluate the regenerators with different geometries [22], where St is Stanton number, Pr is Prandtl number, and f_F is a Fanning friction number. This parameter is a measure of the ratio of heat transfer to pressure loss. Mitchell et al. [23] proposed a “figure of merit” based on experimental data. It measures the heat transfer effectiveness per unit of flow resistance of the regenerators, and is expressed as follows,

$$F_m = \frac{1}{f_D \left(\frac{Pe}{4Nu} + \frac{4Nk}{Pe} \right)}, \quad (1)$$

where f_D is the Darcy friction factor, Pe is Peclet number, Nu is the Nusselt number, and Nk is a measure of thermal dispersion. Based on this figure of merit, their experimental results showed that the

etched stainless steel foil regenerator (similar to parallel-plates) was better than screens or fibers of 70% porosity, or packed spheres of 39% porosity. However, it should be noted that with a cooler, the thermal mass of the regenerators is much more important than that in the engines because of the higher heat capacity of the cold fluid and the lower heat capacity of the regenerator material at low temperatures. Furthermore, the local acoustic impedance of the regenerator plays an important role in the travelling-wave engines. Therefore, it is necessary to include the effect of impedance for the comparison of regenerators.

Generally, pressure and velocity amplitudes and their relative phase angles determine the energy fluxes through the regenerator. The impedance ultimately determines how large these fluxes are. The impedance values are generally determined by the acoustic network which consists of the acoustic compliance, inertance tube and the resistance. According to the literature survey above, it is clear that, unlike in the traditional Stirling engine, the role of the regenerator's flow resistance in the travelling-wave thermoacoustic engine is complicated by the fact that the compact network needs to tailor both the impedance and the time phasing in the regenerator at the same time. It is still not very clear how the impedance affects the performance of the travelling-wave thermoacoustic engine in terms of the power output, onset temperature, and thermal efficiencies. Clearly, for different applications, different performance characteristics are preferred. Furthermore, the role of the flow resistance in the thermoacoustic regenerator also needs further study, together with its relationship with the regenerator impedance.

This paper will focus on the regenerator of the travelling-wave engine, and will analyze the relationship between the acoustic impedance, flow resistance, viscous losses, heat relaxation loss and channel transverse dimensions. Much of the motivation behind the current work is driven by the interest as to whether there is a theoretical “optimum” of these parameters and/or the optimum of their combination for a specific application. Secondly, and more generally, the research attempts to develop a simple but effective methodology for the comparison of regenerators with different geometrical configurations or dimensions. In this work, the regenerators are studied in the travelling-wave for the ideal gas, based on the linear thermoacoustic theory. A pair of non-dimensional factors is obtained from the viewpoint of the “net” acoustic power production within the regenerator. These factors reflect the impact of the channel dimension, local acoustic impedance and the channel geometries on the power output capacity of the regenerators. Using this simple method, regenerators with different dimensions and channel geometries have been investigated numerically and compared with each other.

2. Theoretical analysis

2.1. Regular geometries of regenerators

According to the linear thermoacoustic theory [24,25], the time-averaged acoustic power $d\dot{E}_2$ produced in a length dx of the channel can be written in the complex notation in the general form as

$$\frac{d\dot{E}_2}{dx} = \frac{1}{2} \text{Re} \left[\tilde{U}_1 \frac{dp_1}{dx} + \tilde{p}_1 \frac{dU_1}{dx} \right], \quad (2)$$

where U_1 and p_1 are complex volumetric velocity and pressure, respectively. ‘ $\sim$ ’ indicates a complex conjugate. Subscript 2 indicates that it is a second order quantity. $\text{Re}[\]$ denotes the real part of a complex number. Eq. (2) can be written as [25]

$$\frac{d\dot{E}_2}{dx} = -\frac{r_v}{2}|U_1|^2 - \frac{1}{2r_k}|p_1|^2 + \frac{1}{2}\text{Re}[g\bar{p}_1U_1]. \quad (3)$$

Viscous resistance per unit length of the channel, r_v , thermal-relaxation conductance per unit length of the channel, $1/r_k$, and the complex gain/attenuation constant for the volume flow rate, g , are defined as follows:

$$r_v = \frac{\omega\rho_m}{A_g} \frac{\text{Im}[-f_v]}{|1-f_v|^2}, \quad (4)$$

$$\frac{1}{r_k} = \frac{\gamma-1}{\gamma} \frac{\omega A_g}{p_m} \text{Im}[-f_k], \quad (5)$$

and

$$g = \frac{(f_k - f_v)}{(1-f_v)(1-\sigma)} \frac{1}{T_m} \frac{dT_m}{dx}. \quad (6)$$

f_v and f_k are given in [24] and [25] in detail. γ , σ , ρ_m , p_m and T_m are the ratio of specific heat capacities, Prandtl number, mean density, mean pressure and mean temperature of the working gas, respectively. A_g is the cross-sectional area of gas channels in the regenerator. Furthermore, $A_g = \phi A$, where ϕ and A are porosity and cross-sectional area of the regenerator, respectively.

On the right hand side (RHS) of Eq. (3), the first two terms represent viscous and thermal-relaxation dissipation, respectively, which always consume acoustic power, regardless of the temperature gradient along the length of the regenerator. The third term denotes the acoustic power produced (or consumed) by the regenerator due to axial temperature gradient. It depends on the magnitude and direction of the axial temperature gradient. In the engines, T_m increases in the direction of positive acoustic power flow, so the third term denotes the acoustic power produced from heat energy. For the refrigerators, T_m decreases in the direction of positive acoustic power flow, in which case the third term means the acoustic power consumed to pump heat from the cold to the hot end of the regenerator. This paper focuses only on the analysis of regenerators in engines. Therefore, it will be more convenient to refer to $d\dot{E}_2/dx$ as “net” time-averaged acoustic power production per unit length of the regenerator, because it is a net effect due to the acoustic power dissipation (the first two terms of the RHS of Eq. (3)) and the acoustic power production (the third term).

For the travelling-wave, the phase angle between U_1 and p_1 , $\theta(U_1, p_1) = 0$. Hence

$$\text{Re}[\bar{p}_1U_1] = |p_1||U_1|, \quad \text{Im}[\bar{p}_1U_1] = 0. \quad (7)$$

Substituting Eq. (7) into Eq. (3), the following is obtained:

$$\frac{d\dot{E}_2}{dx} = \frac{1}{2}|U_1|^2 \left\{ -r_v - \frac{1}{r_k} \frac{|p_1|^2}{|U_1|^2} + \text{Re}[g] \frac{|p_1|}{|U_1|} \right\}. \quad (8)$$

In Eq. (8)

$$|p_1|/|U_1| = |p_1/U_1|, \quad (9)$$

where p_1/U_1 is the local acoustic impedance Z of the acoustic wave. For a lossless planar travelling-wave, Z is defined as [24]

$$Z = \frac{p_1}{U_1} = \frac{\rho_m a}{A_g}, \quad (10)$$

where a is the speed of sound. However, for the practical travelling-wave engines, the local acoustic impedance should be much bigger than $\rho_m a/A_g$ to decrease the large viscous losses resulting from high acoustic velocities [9,12]. Here, for convenience it is expressed as

$$Z_{TW} = n \frac{\rho_m a}{A_g}. \quad (11)$$

Subscript “TW” refers to the travelling-wave, n is a real number and denotes a dimensionless local acoustic impedance factor. Substituting Eqs. (4)–(6) and (11) into Eq. (8), the following relationship is obtained:

$$\frac{d\dot{E}_2}{dx} = \frac{1}{2}|U_1|^2 \left\{ -\frac{\omega\rho_m}{A_g} \frac{\text{Im}[-f_v]}{|1-f_v|^2} - \frac{\gamma-1}{\gamma} \frac{\omega A_g}{p_m} \text{Im}[-f_k] \left(n \frac{\rho_m a}{A_g} \right)^2 + \frac{1}{T_m} \frac{dT_m}{dx} \text{Re} \left[\frac{(f_k - f_v)}{(1-f_v)(1-\sigma)} \right] \left(n \frac{\rho_m a}{A_g} \right) \right\}. \quad (12)$$

For ideal gases, the speed of sound $a = \sqrt{\gamma \mathcal{R} T_m}$ ($\mathcal{R}$ is the gas constant per unit mass), and the mean pressure $p_m = \rho_m \mathcal{R} T_m$. In addition, $a = \omega \lambda / 2\pi$, where λ is the wavelength. Thus, Eq. (12) can be simplified to:

$$\frac{d\dot{E}_2}{dx} = \left(\frac{\omega}{2\pi} \frac{\rho_m}{A_g} |U_1|^2 \right) \left\{ -\pi \frac{\text{Im}[-f_v]}{|1-f_v|^2} - \pi n^2 (\gamma-1) \text{Im}[-f_k] + \frac{n}{2} \text{Re} \left[\frac{(f_k - f_v)}{(1-f_v)(1-\sigma)} \right] \frac{dT_m/dx}{T_m/\lambda} \right\}. \quad (13)$$

It can be found that $\frac{\omega}{2\pi} \frac{\rho_m}{A_g} |U_1|^2$ has the same dimension as that of $\frac{d\dot{E}_2}{dx}$. The factor in the curly brackets is dimensionless and in the current analysis is expressed as follows:

$$\Psi_U = -\pi \frac{\text{Im}[-f_v]}{|1-f_v|^2} - \pi n^2 (\gamma-1) \text{Im}[-f_k] + \frac{n}{2} \text{Re} \left[\frac{(f_k - f_v)}{(1-f_v)(1-\sigma)} \right] \Theta_{\nabla T}, \quad (14)$$

where $\Theta_{\nabla T}$ is a dimensionless temperature gradient defined as

$$\Theta_{\nabla T} = \frac{dT_m/dx}{T_m/\lambda}. \quad (15)$$

Therefore, $d\dot{E}/dx$ is proportional to Ψ_U if $\frac{\omega}{2\pi} \frac{\rho_m}{A_g} |U_1|^2$ is given.

On the other hand, substituting Eq. (11) into Eq. (13) yields

$$\frac{d\dot{E}_2}{dx} = \left(\frac{\omega}{2\pi} \frac{A_g |p_1|^2}{\rho_m a^2} \right) \left\{ -\frac{1}{n^2} \pi \frac{\text{Im}[-f_v]}{|1-f_v|^2} - \pi (\gamma-1) \text{Im}[-f_k] + \frac{1}{2n} \text{Re} \left[\frac{(f_k - f_v)}{(1-f_v)(1-\sigma)} \right] \frac{dT_m/dx}{T_m/\lambda} \right\}. \quad (16)$$

Similarly, $\frac{\omega}{2\pi} \frac{A_g |p_1|^2}{\rho_m a^2}$ also has the same dimension as that of $\frac{d\dot{E}_2}{dx}$. The factor in the curly brackets is dimensionless and can be expressed as follows:

$$\Psi_p = -\frac{1}{n^2} \pi \frac{\text{Im}[-f_v]}{|1-f_v|^2} - \pi (\gamma-1) \text{Im}[-f_k] + \frac{1}{2n} \text{Re} \left[\frac{(f_k - f_v)}{(1-f_v)(1-\sigma)} \right] \Theta_{\nabla T}, \quad (17)$$

Here, we call Ψ_U and Ψ_p “power factors” because they reflect the power output capacity of the regenerators. One can find that

$$\Psi_p = \frac{\Psi_U}{n^2}. \quad (18)$$

According to the derivation above and Ref. [24], it can be found that Ψ_p (or Ψ_U) is a function of f_k , f_v , n , σ , and γ . For regular geometries, f_v and f_k are analytical functions of δ_k/r_h . σ and γ are given when the working gas is given. Assuming the temperature distribution is linear along the length of the regenerator, the mean value of $\Theta_{\nabla T}$ can be estimated from Eq. (15) by using the total length of the regenerator, the temperature difference between the two ends and the mean temperature of the gas in the middle of the stack (T_m). Therefore, the impact of δ_k/r_h and n on $d\dot{E}/dx$ can be studied numerically.

Compared with Eq. (8), it can be found that, in the RHS of Eqs. (14) and (17), the first and second terms denote dissipation by viscosity and thermal relaxation, respectively. The third term represents the acoustic power produced from heat energy. Qualitatively it can be concluded that, the acoustic impedance reflects the proportion among the viscous dissipation, thermal-relaxation dissipation, and the new acoustic power production from heat energy. According to Eqs. (14) and (17), the weighting of the third term is much bigger than that of the first term, because $\Theta_{\nabla T} \gg 1$ and $n \gg 1$. Therefore, one can see that increasing volumetric velocity increases the first term, which means viscous dissipation, however, it also increases the third term which corresponds to the acoustic power production.

However, it should be noted that, in practical travelling-wave engines, $|U_1|$, $|p_1|$ and n are determined by the network of all the acoustic components which is often quite complex. One cannot arbitrarily increase or decrease n whilst keeping $|U_1|$ or $|p_1|$ constant. Nevertheless, one could focus on the analysis of how the transverse dimension and impedance impact the acoustic power flow in the regenerators. Therefore, Eqs. (14) and (17) will give a useful insight into the regenerator's power output capacity.

2.2. Stacked-screen regenerators

For most practical travelling-wave thermoacoustic systems, the regenerator comprises of layers of stacked screens (wire-meshes). It is possible to perform the analysis similar to that outlined in Section 2.1, providing appropriate analytical expressions can be found. Swift and Ward [27] proposed a set of thermoacoustic equations for stacked-screen regenerator with the method of simple harmonic analysis. Based on their equations, the analysis of the acoustic power dissipation and production in the stacked-screen regenerator can be developed further. Substituting their continuity and momentum equation [27] into Eq. (3) yields

$$\frac{d\dot{E}_2}{dx} = \frac{1}{2} A_g \text{Re} \left\{ i\omega \left[\beta_m \frac{T_m \beta_m}{\rho_m c_{p,m}} \frac{\varepsilon_s + (g_c + e^{2i\theta_p} g_v) \varepsilon_h}{1 + \varepsilon_s + (g_c + e^{2i\theta_T} g_v) \varepsilon_h} - \frac{\gamma_m}{\rho_m a_m^2} \right] |p_1|^2 \right. \\ \left. - i\omega \rho_m |\langle u_1 \rangle|^2 - \frac{\mu_m}{r_h^2} \left[\frac{c_1(\phi)}{8} + \frac{c_2(\phi) Re_1}{3\pi} \right] |\langle u_1 \rangle|^2 \right. \\ \left. + \beta_m \frac{dT_m}{dx} \left[1 - \frac{\varepsilon_s + (g_c - g_v) \varepsilon_h}{1 + \varepsilon_s + (g_c + e^{2i\theta_T} g_v) \varepsilon_h} \right] \tilde{p}_1 \langle u_1 \rangle \right\}. \quad (19)$$

In Eq. (19), $\langle \rangle$ denotes a local spatial average. β_m , γ_m and μ_m are mean thermal expansion coefficient, mean ratio of specific heats, and mean dynamic viscosity of the gas, respectively. $b(\phi)$, $c_1(\phi)$, $c_2(\phi)$, $g_c(Re_1)$, and $g_v(Re_1)$ are factors resulting from the fitting of data of Kays and London [27,28]. θ_p and θ_T are defined as $\theta_p = \theta(p_1, \langle u_1 \rangle)$ and $\theta_T = \theta(\langle u_1 \rangle, \langle T \rangle_{u,1})$, respectively. $\varepsilon_s = \phi \rho_m c_{p,m} / (1 - \phi) \rho_{s,m} c_{s,m}$ (subscript s means solid) gives the ratio of gas heat capacity to solid heat capacity in the stacked-screen regenerator. ε_h is defined as $\varepsilon_h = 8ir_h^2/b(\phi)\sigma^{1/3}\delta_k^2$. Reynolds number $Re_1 = 4|\langle u_1 \rangle| r_h \rho_m / \mu_m$ is a positive real number. More detailed description of these variables and definitions is given in Ref. [27].

By analogy with Eq. (11), the effective local specific acoustic impedance can be introduced as follows:

$$z_e = p_1 / \langle u_1 \rangle = n \rho_m a. \quad (20)$$

Then, assuming a pure travelling-wave and ideal gas, one can obtain Eqs. (21)–(23) for stacked-screen regenerators through similar derivations as those for the regular geometries

$$\frac{d\dot{E}_2}{dx} = \left(\frac{\omega}{2\pi} A_g \rho_m |\langle u_1 \rangle|^2 \right) \Psi_{p,SCR}, \quad (21)$$

where

$$\Psi_{U,SCR} = -\frac{\pi\sigma}{2} \left(\frac{\delta_k}{r_h} \right)^2 \left(\frac{c_1(\phi)}{8} + \frac{c_2(\phi) Re_1}{3\pi} \right) - \pi n^2 (\gamma - 1) \\ \times \text{Im} \left(\frac{\varepsilon_s + (g_c + g_v) \varepsilon_h}{1 + \varepsilon_s + (g_c + e^{2i\theta_T} g_v) \varepsilon_h} \right) \\ + \frac{n}{2} \text{Re} \left(1 - \frac{\varepsilon_s + (g_c - g_v) \varepsilon_h}{1 + \varepsilon_s + (g_c + e^{2i\theta_T} g_v) \varepsilon_h} \right) \Theta_{\nabla T}, \quad (22)$$

Here, subscript “SCR” indicates a stacked-screen regenerator. An analogous equation to (17) has the following form:

$$\frac{d\dot{E}_2}{dx} = \left(\frac{\omega}{2\pi} \frac{A_g |p_1|^2}{\rho_m a^2} \right) \Psi_{p,SCR}. \quad (23)$$

where

$$\Psi_{p,SCR} = -\frac{\pi\sigma}{2n^2} \left(\frac{\delta_k}{r_h} \right)^2 \left(\frac{c_1(\phi)}{8} + \frac{c_2(\phi) Re_1}{3\pi} \right) \\ - \pi (\gamma - 1) \text{Im} \left(\frac{\varepsilon_s + (g_c + g_v) \varepsilon_h}{1 + \varepsilon_s + (g_c + e^{2i\theta_T} g_v) \varepsilon_h} \right) \\ + \frac{1}{2n} \text{Re} \left(1 - \frac{\varepsilon_s + (g_c - g_v) \varepsilon_h}{1 + \varepsilon_s + (g_c + e^{2i\theta_T} g_v) \varepsilon_h} \right) \Theta_{\nabla T}. \quad (24)$$

Here, $\Psi_{U,SCR}$ and $\Psi_{p,SCR}$ are the pair of “power factors” for stacked-screen regenerators. Similarly, it can be found that

$$\Psi_{p,SCR} = \Psi_{U,SCR} / n^2. \quad (25)$$

According to the derivation above and Ref. [27], it can be found that $\Psi_{p,SCR}$ (or $\Psi_{U,SCR}$) is a function of δ_k/r_h , n , ϕ , Re_1 , $\Theta_{\nabla T}$, σ , γ , ε_s and θ_T . Furthermore, for practical travelling thermoacoustic engine, the impact of ε_s and θ_T on $\Psi_{U,SCR}$ can be neglected [27]. σ and γ are given when the working gas is given. Therefore, the impact of δ_k/r_h and n on $d\dot{E}_2/dx$ can be studied when ϕ , Re_1 and $\Theta_{\nabla T}$ are given.

3. Numerical analysis

Theoretically, both Ψ_p and Ψ_U could be used for the analysis of the regenerators. However, in practice, it is more convenient to measure $|p_1|$ than $|U_1|$. Therefore, Ψ_p and $\Psi_{p,SCR}$ are used in the analysis below, where numerical calculations for parallel-plate and stacked-screen regenerators have been performed. The dimensionless transverse dimension of the channel, δ_k/r_h , was used for the analysis. All calculations were performed for helium as the working fluid ($\sigma = 2/3$, $\gamma = 5/3$).

According to Eqs. (17) and (24), $\Theta_{\nabla T}$ is a required value for the numerical analysis. As shown in Eq. (15), $\Theta_{\nabla T}$ can be estimated by using the length of the regenerator instead of dx , and the temperature difference between the two ends instead of dT_m with an additional assumption of a linear temperature gradient along the regenerator. The wavelength is decided by the operating frequency (usually several resonator lengths in practical systems). Therefore, one can obtain the approximate mean value of $\Theta_{\nabla T}$ in the regenerator. For example in Backhaus and Swift's travelling-wave engine [7], at its point of the highest power production, the hot and cold end temperatures of the regenerator are 725 °C and 40 °C, respectively. The length of the regenerator is 7.3 cm. The operating frequency is 84 Hz with helium at a mean pressure of 31 bars as the working gas. Based on this information, mean value of $\Theta_{\nabla T}$ is estimated as about 172 in the regenerator. Here, for the convenience of further comparison with these experimental results, $\Theta_{\nabla T} = 172$ is set for the calculations.

3.1. Parallel-plate regenerator

Fig. 2 shows the calculated Ψ_p for the parallel-plate regenerator when δ_k/r_h varies, for several values of n treated as a parameter. There is a maximum value of Ψ_p for each n when δ_k/r_h varies,

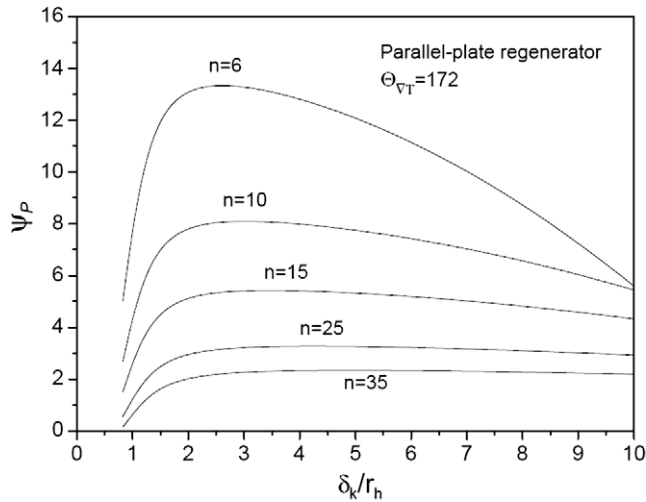


Fig. 2. Parallel-plate regenerator in the travelling-wave: the calculated value of Ψ_p versus δ_k/r_h for different local acoustic impedances. For each n , there is a maximum value of Ψ_p when δ_k/r_h varies.

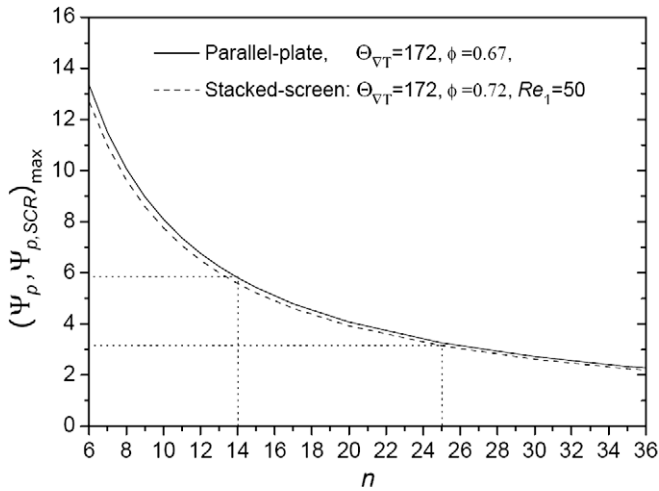


Fig. 3. The relationship of $(\Psi_p)_{\max}$ versus n for both the parallel-plate and stacked-screen regenerator. The two plotted points schematically show the operating states of the two regenerators tested by Backhaus and Swift [7,20].

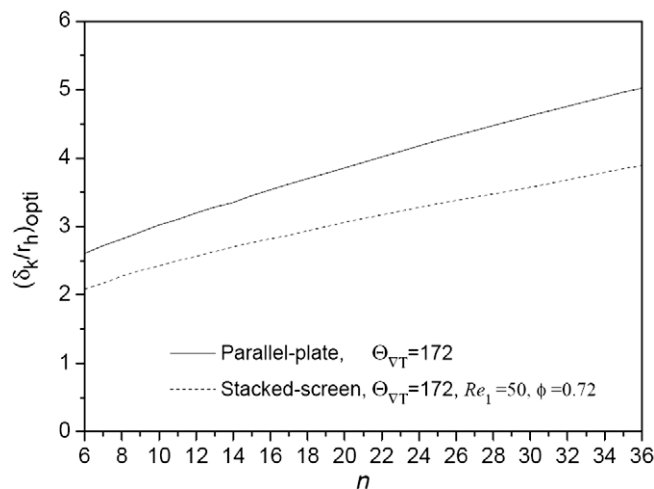


Fig. 4. The calculated relationship between $(\delta_k/r_h)_{\text{opti}}$ and n for the tested parallel-plate and stacked-screen regenerators.

designated as $(\Psi_p)_{\max}$. This corresponds to the optimum value of δ_k/r_h , denoted as $(\delta_k/r_h)_{\text{opti}}$. On the other hand, one can find that the optimal range of δ_k/r_h becomes broader as n increases. For example for $n = 25$ and $n = 35$, Ψ_p increases sharply when $\delta_k/r_h < 2.5$, but for $\delta_k/r_h > 3$ the dependence is rather weak. This means that in practical travelling-wave thermoacoustic engines, a broad range of δ_k/r_h may be suitable for regenerators. Furthermore, Ψ_p decreases rapidly as n increases if δ_k/r_h is fixed. Because Ψ_p is related to the net time-averaged acoustic power production of the unit length of the regenerator, this figure reveals that both the local acoustic impedance and channel dimension significantly influence the acoustic power output of regenerators in the travelling-wave engines.

In addition, Fig. 2 can be interpreted as a graph of $(\Psi_p)_{\max}$ and $(\delta_k/r_h)_{\text{opti}}$ as functions of n (see solid lines in Figs. 3 and 4). In Fig. 3, the solid line shows that $(\Psi_p)_{\max}$ is nearly in inverse proportion to n . This agrees with the analysis in Section 2.1: the third term in Eq. (17), which has the biggest weighting, is indeed inversely proportional to n . Hence, if $\frac{\omega}{2\pi} \frac{A_g |p_1|^2}{\rho_m a^2}$ is given, the acoustic power output of the regenerator is in practice almost inversely proportional to impedance.

Fig. 4 shows the relationship of $(\delta_k/r_h)_{\text{opti}}$ versus n . It can be seen that, $(\delta_k/r_h)_{\text{opti}}$ slightly increases as n increases. Clearly, the optimum dimensionless channel dimension in the regenerator exists; furthermore, it depends on the local impedance. As the impedance increases, the corresponding optimum $(\delta_k/r_h)_{\text{opti}}$ increases, i.e. the optimum traverse dimension of channel decreases. Theoretically, it is possible to obtain an optimum combination between the transverse dimension r_h of channel and impedance, which would lead to the highest acoustic power output.

3.2. Stacked-screen regenerator

Eq. (24) is used for the analysis of the stacked-screen regenerators. Compared with those for parallel-plate regenerator, two additional parameters, ϕ and Re_1 should be provided for the calculations. To compare with the results of Backhaus and Swift's travelling-wave engine (mean pressure of 30 bars) with the stacked-screen regenerator [7], Θ_{vT} , ϕ and Re_1 are set according to its highest power operating point. In this case, Θ_{vT} is also estimated at about 172, and the measured porosity $\phi = 0.72$. One can estimate the mean value of Re_1 at about 50 in the regenerator based on DeltaE [29] simulations.

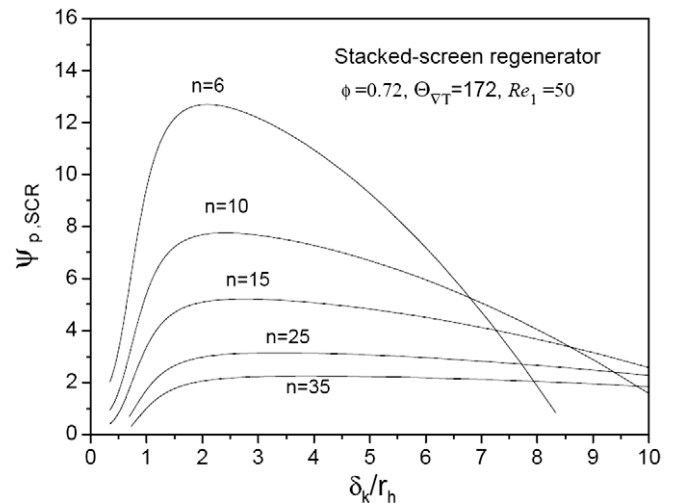


Fig. 5. Stacked-screen regenerator in the travelling-wave: the calculated values of $\Psi_{p,SCR}$ versus δ_k/r_h for different local acoustic impedances. For each n , there is a maximum value of $\Psi_{p,SCR}$ when δ_k/r_h varies.

The calculated results are quite similar to those for the parallel-plate regenerator and shown in Figs. 3–5. Firstly, Fig. 5 shows the calculated $\Psi_{p,SCR}$ for the selected stacked-screen regenerator when δ_k/r_h varies, for several values of n . The optimal points in Fig. 5 are also shown as dotted lines in Figs. 3 and 4, respectively.

From Figs. 3–5, it can be found that the local acoustic impedance and channel dimensions influence the power output of stacked-screen regenerator almost in the same way as those for the parallel-plate regenerator. The method developed for the regular geometrical configuration is applicable to the stacked-screen regenerators.

3.3. Comparisons with experimental results

As already mentioned, the regenerators discussed above have the same dimensions as those studied by Backhaus and Swift [7,20]. Their experimental results indicate that, under nearly the same operating conditions, the maximum acoustic power output of the travelling-wave engine with the parallel-plate regenerator is almost double of that for the engine with the stacked-screen regenerator. However, according to Fig. 3, one can see that the difference between $(\Psi_p)_{\max}$ and $(\Psi_{p,SCR})_{\max}$ is very small (about 4%) for each given n . The reason behind such a large difference in power output needs to be identified.

For further analysis, DeltaE [29] code is used to simulate the whole engine of the two cases tested by Backhaus and Swift [7,20], based on the published experimental results. Dividing the regenerator into 16 segments, the distribution of pressure amplitudes $|p_1|$ and volumetric velocity $|U_1|$ are obtained and shown in Fig. 6a and b, respectively. In Fig. 6a, it can be found that $|p_1|$ decreases along the length of the regenerators from the cold to hot end. Furthermore, the pressure amplitude drops $|\Delta p_1|$ are about 0.3 and 0.15 bar in the tested stacked-screen and parallel-plate regenerators, respectively. In Fig. 6b, it can be seen that the volumetric velocity $|U_1|$ increases along the length of the regenerators from the cold to hot end. Here, one can find that the temperature gradient has a large effect on the volumetric velocity. Furthermore, $|U_1|$ through the parallel-plate regenerator is about 1.5 times larger than that through the tested stacked-screen regenerator at each cross section. Based on the simulated results shown in Fig. 6a

and b, one can calculate the approximate dimensionless impedance factor n in the two tested regenerators using Eqs. (11) and (20), as shown in Fig. 6c. It can be seen that n decreases along the length of the regenerator from the cold to the hot end. Furthermore, the mean value of dimensionless impedance factor n in the stacked-screen regenerator is about 1.7 times of that in the tested parallel regenerator.

Using the above simulation results, the viscous dissipation in the tested stacked-screen regenerator is estimated as about 40 W more than that in the parallel-plate regenerator. Therefore, the viscous dissipation difference could be partly a reason behind the large output difference. However, this is still not large enough to explain the experimental differences.

Fig. 6d shows the obtained phase angle between U_1 and p_1 through the regenerators. It can be seen that, in the two regenerators, the phase angles $\theta(U_1, p_1)$ are a bit different near the cold end, but quite similar near the hot end of the regenerators. This difference is mainly due to the different resistance in the two regenerators. Given that the majority of the acoustic power is produced near the hot end of the regenerators, the difference of the phase angle shown in Fig. 6d will also account for some of the difference in the acoustic power outputs of these test cases, but clearly it is still not the key reason.

According to Fig. 6c, the estimated values of n close to the middle of the regenerators are about 14 and 25 in the parallel-plate and stacked-screen regenerators, respectively. Going back to Fig. 3, the two tested regenerators by Backhaus and Swift [7,20] can be schematically shown by the two plotted points. One can find the corresponding $(\Psi_p)_{\max} \approx 5.9$ and $(\Psi_{p,SCR})_{\max} \approx 3.1$, which indicates that the possible maximum capacity of acoustic power output for the parallel-plate regenerator is about twice of that from the stacked-screen regenerator. It is quite interesting that the obtained result agrees with the experimental results qualitatively [7,20], although this kind of analysis is rather approximate, because it is based on an ideal gas and a pure travelling-wave. Therefore, it can be qualitatively concluded that the difference of the impedances is the possible key reason behind the large difference in power outputs for the two tested cases.

3.4. The role of the viscous resistance in the regenerator

At this point, the relationship between flow resistance and impedance can be discussed further. In the same notation as Eq. (4), the viscous resistance per unit length of stacked-screen regenerator is [25,27]

$$r_{v,SCR} = \frac{\mu_m}{r_h^2 \phi A} \left(\frac{c_1(\phi)}{8} + \frac{c_2(\phi) Re_1}{3\pi} \right). \quad (26)$$

Substituting the viscous penetration depth $\delta_v = \sqrt{2\mu_m/\omega\rho_m}$ into Eq. (26) to cancel μ_m leads to

$$r_{v,SCR} = \frac{1}{2} \frac{\omega\rho_m}{\phi A} \left(\frac{\delta_v}{r_h} \right)^2 \left(\frac{c_1(\phi)}{8} + \frac{c_2(\phi) Re_1}{3\pi} \right). \quad (27)$$

Based on the parameters used in Sections 3.1 and 3.2, one can estimate the mean resistance along the length of the two regenerators by using Eqs. (4) and (27). The mean value of $r_{v,SCR}/r_v$ in the regenerators is about 2.8. So the viscous resistance of the stacked-screen regenerator is substantially bigger than that of parallel-plate regenerator.

On the other hand, neglecting the compliance effect due to regenerator's porosity, the pressure amplitude drop through the regenerator [7]

$$\Delta p_1 \propto \frac{\tau + 1}{2} R_v U_{1,c}. \quad (28)$$

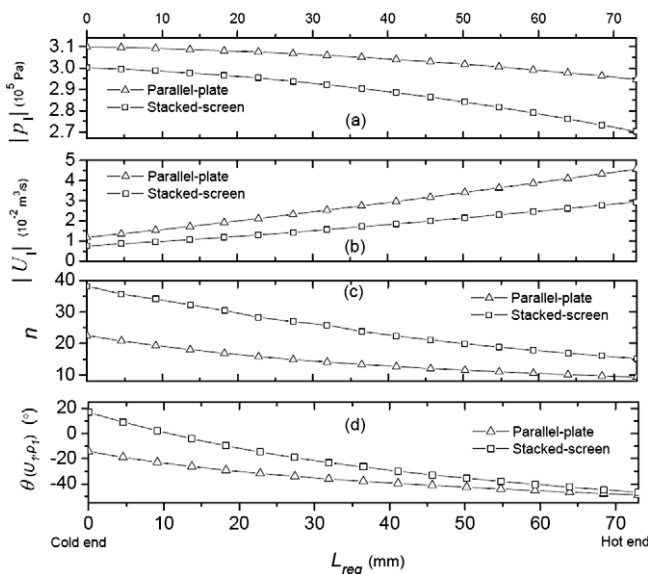


Fig. 6. Results of DeltaE simulations, based on the experimental data of Backhaus and Swift [7,20]. Distributions of $|p_1|$, $|U_1|$, n and $\theta(U_1, p_1)$ through the two tested regenerators.

Here, $\tau = T_h/T_c$, the subscripts h and c mean the hot and cold end of the regenerator, and R_v is the viscous resistance of the regenerator. Therefore, according to Eq. (28), it can be seen that higher viscous resistance R_v (or r_v) leads to a smaller $|U_1|$, and thus will lead to a high impedance when the pressure amplitude drop Δp_1 is given. This analysis is consistent with the simulation results in Fig. 6.

As discussed in the introduction, the thermoacoustic core in such travelling-wave engines is an acoustic power amplifier. This amplification of acoustic power can be approximately expressed as,

$$E_{out} = \tau E_{in}, \quad (29)$$

where E_{in} and E_{out} are the acoustic power that runs into the ambient end of the regenerator, and the acoustic power runs out of the hot end of the regenerator, respectively. Therefore, the total net acoustic power produced by the regenerator can be defined as [7]

$$E_{net} = (\tau - 1)E_{in}. \quad (30)$$

For a given pressure amplitude at the ambient end of the regenerator $p_{1,in}$, W_{in} can be approximately expressed as,

$$E_{in} = \frac{|p_{1,in}|^2}{2\text{Re}[Z_{in}]}, \quad (31)$$

Assuming a travelling-wave phase at the ambient end of the regenerator, substituting Eq. (11) into Eq. (31) leads to (cf. [19])

$$E_{in} = \frac{A_g |p_{1,in}|^2}{2n\rho_m a}. \quad (32)$$

By combining Eqs. (32) and (30), the relationship between the impedance and the net acoustic power production can be obtained as

$$E_{net} = \frac{(\tau - 1)A_g |p_{1,in}|^2}{2n\rho_m a}. \quad (33)$$

Eq. (33) indicates that, if the operating conditions and the pressure amplitude are given, the net power production in the regenerator is almost inversely proportional to the impedance factor n , which agrees with Eqs. (17) and (24), and the results shown in Fig. 3.

Qualitatively, it can be concluded that a higher viscous resistance of the stacked-screen regenerator leads to a smaller volumetric velocity, and consequently, also a higher impedance, in the tested stacked-screen regenerator [7] compared with the parallel-plate regenerator [20]. A smaller volumetric velocity and a higher viscous dissipation both result in a smaller acoustic power output for the travelling-wave engine with the stacked-screen regenerator [7]. But, the smaller volumetric velocity (i.e. higher impedance as shown in Fig. 3) is the key reason because it decreases the power input fed back to the cold end of the stacked-screen regenerator [7] compared with the parallel-plate one [20].

It should be noted that the current analysis does not account for any nonlinear viscous resistance in the stacked-screen regenerator at high acoustic amplitudes. If this nonlinear resistance were considered, $r_{v,SCR}/r_v$ would increase. Thus, the volumetric velocity would decrease in the stacked-screen regenerator, and subsequently the impedance would increase. As a result, the predicted difference of power output capacity as shown in Fig. 3 would be higher.

4. Discussion and conclusions

Based on the assumptions of the pure travelling-wave and ideal gas, a pair of dimensionless factors, Ψ_U and Ψ_p , have been obtained to evaluate the acoustic power output capacity of regenerators in

the travelling-wave thermoacoustic engines. These factors can clearly reveal the relationship between the flow resistance, impedance, and channel dimensions with the acoustic power output capacity.

It is shown that the local acoustic impedance of the regenerators essentially represents the proportion between the acoustic power produced from the heat energy through the thermoacoustic processes and the acoustic power dissipated by viscous and thermal-relaxation effects in the regenerators. The net acoustic power output of the regenerator is determined by its impedance. This is because the impedance determines the amount of power fed into the ambient end of the regenerator. For a given pressure amplitude, a higher impedance leads to a lower acoustic power input. Therefore, the local acoustic impedance has to be taken into account for any evaluation of regenerators, as well as the comparison between different geometrical configurations or dimensions.

For a given pressure amplitude, $|p_1|$, increasing the impedance will decrease the volumetric velocity, and thus will decrease the acoustic power output of the engine. Therefore, relatively low impedance is preferred for the travelling-wave thermoacoustic engines to obtain a high acoustic power output. Certainly, higher volumetric velocity will cause higher dissipation if the resistance is not reduced, and then degrade the thermodynamic efficiency. There should be a compromise between high efficiency and high power output according to the requirements of different applications.

The resistance of the regenerator plays the key role in determining the regenerator's impedance. For a given pressure amplitude, the higher resistance will decrease the volumetric velocity through the regenerator, which consequently causes the high impedance of the regenerator. This is the key reason of the large difference of power output for the tested stacked screen [7] and parallel-plate [20] regenerators. In any case, decreasing the flow resistance is most crucial in the practical design of regenerators for travelling-wave thermoacoustic engines. Practically, one can reduce the regenerator's resistance by using regular configurations such as parallel-plate to achieve relatively low resistance, reducing the length of the regenerator, or increasing the cross-sectional area.

Using the same methodology as in Section 3.1, one can perform a similar numerical analysis for pin-array and circular-pores regenerators. Obtaining similar results to those shown in Figs. 2–4 is relatively straightforward for these two geometrical configurations and therefore has been omitted in this paper. Compared with parallel-plate regenerator with the same porosity, it is found that for given impedances these regenerators have almost the same possible maximum acoustic power output capacity as those of parallel-plate regenerator. In other words, from the point of view of Rott's function f_k , the geometrical configuration is not a sensitive factor to regenerator's performance for a given impedance. This is mainly due to the fact that for all these geometrical configurations $\text{Re}[f_k]$ approaches 1 when δ_k/r_h is high [25]. However, the regenerator's resistance changes this picture. As shown in Fig. 3, for given impedance, the difference in Ψ_p between the stacked screen and the parallel-plate regenerator is quite small. But different impedance due to different resistance caused large differences in power output. Therefore, it can be inferred that, regardless of the geometric configuration, the regenerator with lower resistance will have a better performance when the other parameters (i.e. porosity, channel dimensions, material, etc.) are the same.

This mechanism is different from that behind the large efficiency difference due to different geometries of stacks in standing-wave engines. It was reported that, among the three regular geometric configurations mentioned above, the pin-array stack is the most efficient one, parallel-plate is the moderate, and the circular-pore stack is the worst one [26,30]. For the stacks, the performance difference is mainly due to the fact that $\text{Im}[f_k]$ is quite

different when $\delta_k/r_h \sim 1$. In contrast to the regenerators, the stacks' impedance is determined by its location in the standing-wave resonator rather than its resistance.

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